

The Least *At Least* Can Do

Klinedinst (in progress) proposes that disjunctions in the scope of universal quantifiers trigger an embedded scalar implicature, as schematized, slightly simplified, in (1). Arguably, this schema is a grammaticalized version of the Gricean implicature in (2).

In this paper I show that we can derive a number of unexplained readings for modal sentences with *at most* and *at least* from adopting these implicature schemata, plus the assumption that *at most/least n* are treated as *exactly n or more than n* and *exactly n or less than n*, respectively.

We start by observing that (3) and (4) together intuitively characterize how much John must weigh in order to start as a bantam weighter: between 116–122lbs. Their truth conditions, however, would make them eligible to describe a situation in which John *de facto* needs to weigh between 118–120lbs; that is, they fail to encode that it is *sufficient* for John to weigh at least 116lbs and at most 122lbs in order to be a bantam weighter. The embedded implicature remedies that, since it adds as an implicature to (3) that (i) in some permitted worlds, John weighs exactly 116lbs, (ii) in some permitted worlds, he weighs more than 116lbs; and to (4) that (iii) in some permitted worlds he weighs 122lbs, and (iv) in some permitted worlds he weighs less than 122lbs.

Second, it should be noted that (3) and (4) have a second reading, which I call the *speaker insecurity* reading; if I claim that John can be a bantam weighter even at 112lbs, you can correct me by uttering (3); crucially, you could do so even if the minimum weight requirement was in fact 120lbs. This reading of (3) is paraphrased in (5). Likewise, you can use (4) to correct my claim that John needs to weigh 125lbs to be super-bantam weighter, as long as the actual minimum weight requirement for that class is somewhere below 122lbs, cf. the paraphrase in (6).

One suspects (correctly, I will argue) that these readings involve scoping *at most/least* above the modal, as in (7). However, as observed for similar data in (Hackl, 2000, ch.3), the two scopings for *at least* turn out to be equivalent: John mustn't weigh less than 116lbs. We claim nevertheless that the speaker insecurity reading (5) does result from scoping *at least* high, but that the difference solely resides in implicatures: the lack of an embedded implicature (since the putative disjunction is no longer in the scope of the modal), plus the addition of the parallel global Gricean implicature (2), cf. (8) (note that on the standard semantics for gradable adjectives (e.g. Heim (2000)) assumed here, $\max(\lambda d. \forall w \in \text{Must}(@)[d \leq \text{John's weight in } w])$, the maximum of the weights John has in every permitted world, comes out to be John's *minimum* required weight).

Similar the truth conditions for *at most* in structure (7), while different from the *in situ* reading are appropriately strengthened using (2) to yield the meaning in (6). The paper proceeds to discuss further data (and puzzling readings) such as (9)–(11), involving possibility, rather than necessity, modals, as well as the quantifiers *more/less than n* (which, it should be noted, are denotationally equivalent to *at least n+1/at most n-1*).

- (1) $\forall x[p(x) \vee q(x)]$ implicates $\exists x[p(x)]$ and $\exists x[q(x)]$
- (2) If a speaker utters *p or q*, it is implied that in all of the speakers doxastic alternatives $q \vee p$, and in some *p*, and in some *q*.
- (3) John must weigh at least 116lbs.
'in all permitted worlds, John weighs exactly 116lbs or he weighs more'
- (4) John must weigh at most 122lbs.
'in all permitted worlds, John weighs exactly 122lbs or he weighs less'
- (5) the weight that John has in every permitted world (formally: $\max(\lambda d. \forall w \in \text{Must}(@)[d \leq \text{John's weight in } w])$) is 116lbs, or it is more
- (6) the weight that John has in every permitted world is 122lbs, or it is less
- (7) [at least/most 118lbs]₁ John must weigh t_1
- (8) in all doxastic alternatives, the required minimum weight for John is 116lbs, or it is more than 116lbs, (quality) and
 - a. in some doxastic alternative, the required minimum weight is 116lbs (scalar), and
 - b. in some doxastic alternative, the required minimum weight is more than 116lbs (scalar)
- (9)
 - a. John is allowed to weigh at least 116lbs.
 - b. John is allowed to weigh at most 122lbs.
- (10)
 - a. John has to weigh more than 115lbs.
 - b. John has to weigh less than 123lbs.
- (11)
 - a. John is allow to weigh more than 115lbs.
 - b. John is allowed to weigh less than 123lbs.

References

- Hackl, Martin. 2000. Comparative quantifiers. Ph.D. thesis, MIT.
- Heim, Irene. 2000. Degree Operators and Scope. Proceedings of SALT X. Ithaca, NY: CLC Publications.
- Klinedinst, Nathan. in progress. Plurals, modals, and conjunctive disjunction. Ph.D. thesis, UCLA.