A Million Monkeys

Daniel Bruhn

30th September 2005

1 The Issue

One often hears the assertion that, given enough time, a million monkeys banging away on typewriters will produce the complete works of Shakespeare. What follows is an attempt to evaluate a greatly-simplified version of this problem.

2 The Problem

Given 1 million typing monkeys, how much time is required for one of those monkeys to produce this sentence?

> The quick brown fox jumped

We will operate under this set of generous assumptions:

- Uppercase/lowercase distinctions and spaces are ignored. Thus, the actual 22-character string desired is: THEQUICKBROWNFOXJUMPED
- All 26 alphabetical character keys have an equal probability of being pressed.
- Each monkey types at a constant rate of 100 wpm.
- Given an average English word length of 9 characters, this yields a monkey typing rate of 900 cpm.
- Said monkeys are not preoccupied with flinging of poo.

3 The Analysis

We begin this analysis by evaluating the performance of one monkey, whom we will name "Bob." The probability of any one alphabetical character key out of the 26 being pressed is $\frac{1}{26}$. Thus, the probability that Bob hits the character T, being the first letter in the sequence THEQUICKBROWNFOXJUMPED, is $\frac{1}{26}$. The probability that Bob first hits T, then H, is therefore $\left(\frac{1}{26}\right) \times \left(\frac{1}{26}\right)$ or $\left(\frac{1}{26}\right)^2$. The probability of T, then H, then E, is $\left(\frac{1}{26}\right)^3$, and given the whole 22 letter sequence we derive this result:

$$\mathbf{P}(THEQUICKBROWNFOXJUMPED) = \left(\frac{1}{26}\right)^{22} = \frac{1}{26^{22}}$$

Because the probability of a coin toss yielding *heads* is $\frac{1}{2}$, we know that the average number of coin tosses we need before we get *heads* is the reciprocal of P(*heads*), i.e. $\frac{1}{\mathbf{P}(heads)} = \frac{1}{\frac{1}{2}} = 2\,\text{tosses}$. The same is true of the alphabetical characters. The number of keystrokes needed before THEQUICKBROWNFOXJUMPED arises is simply the reciprocal of the character sequence's probability:

$$\frac{1}{\mathbf{P}(THEQUICKBROWNFOXJUMPED)} = \frac{1}{\frac{1}{26^{22}}} = 26^{22} \approx 1.347{\times}10^{31}\,\text{keystrokes}$$

Given that monkey Bob types at 900 keystrokes per minute, 60 minutes is 1 hour, 24 hours is one day, and 365.25 days is 1 year, the length of time required for Bob to produce THEQUICKBROWNFOXJUMPED is:

$$26^{22}\,\text{keys} \times \frac{1\,\text{min}}{900\,\text{keys}} \times \frac{1\,\text{hr}}{60\,\text{min}} \times \frac{1\,\text{day}}{24\,\text{hrs}} \times \frac{1\,\text{yr}}{365.25\,\text{days}} \approx 2.846 \times 10^{22}\,\text{years}$$

This value is 28.46 sextillion years, or 28.46 trillion billion years. Accounting for the other 999,999 monkey friends of Bob yields:

$$\frac{2.846 \times 10^{22}\,\text{years}}{10^6} = 2.846 \times 10^{16}\,\text{years}$$

Thus, one million monkeys on typewriters would require **28.46 quadrillion years** (28.46 million billion) to produce THEQUICKBROWNFOXJUMPED.

4 The Conclusion

According to modern scientists, the ages of the universe and of the earth are approximately 14 billion and 4.6 billion years, respectively. But if one million monkeys with typewriters need **28.46 million billion** years to produce a simple 5-word sentence, how could complex human life develop in such a short time?